Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Linear Algebra-II

Subject Code : 4SC04MTC2 **Branch : B.Sc.(Mathematics,Physics)**

Semester : 4 **Date : 12/05/2016** **Time : 2:30 To 5:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define : Orthogonal vectors. (1)
b) Find inner product of (2,-1,6) and (1,1,2) . (1)
c) What is orthogonal linear transformation? (1)
d) True/false: Every orthogonal vectors are linearly independent. (1)
e) Define : Symmetric linear transformation. (1)
f) Write the standard form of Ellipsoid . (1)
g) What is characteristic root of linear transformation T? (1)
h) True/false: If A is symmetric matrix then it has one real Eigen value (1)
i) What do you mean by Conics and Quadrics ? (1)
j) Define: $W^{\perp}$. (1)
k) Write orthonormal basis of R^3 . (1)
l) $W \cap W^{\perp}$ = ---------- . (1)
m) True/false: $\|x\|=\|y\|$ then $x - y \perp x + y$. (1)
n) If u=(3,-4,0) , find $\|u\|$. (1)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If $x \perp y$ and $x \perp z$,show that $x \perp (\alpha y + \beta z)$ for all $\alpha , \beta \in R$. **(2)**
b) What is inner product space? Find angle between (-6, 4 , 6 , 7) and (1 , 6, 4 ,-6) . **(4)**
c) State and prove Riesz-representation theorem . **(8)**

Q-3 **Attempt all questions** **(14)**

a) Apply Gram-schmidth process to obtain orthonormal set **(7)**

$\{(1,-1,1,1),(0,-1,0,2),(-1,1,1,-1)\}$ in R^4.

b) Show that parallelogram is rhombus if and only if the diagonals are perpendicular to each other **(7)**

Q-4 **Attempt all questions** **(14)**

a) If V is vector space and W is any subset of V then show that $W^{\perp}$is subspace of V. **(3)**

b) Find the angle between f and g where f(t) = t and g= h-3⟨h, f⟩f , h(t) = t^2. **(4)**

c) Using gram-schmidth process obtain orthonaormal set for{1, t , t^2} **(7)**

of with inner product ⟨p, q⟩=$\int_0^1 p(t)q(t)dt$.

Q-5 **Attempt all questions** **(14)**

a) Prove that $(W^{\perp})^{\perp}$ = W . **(2)**

b) Prove that a parallelogram is rectangle iff the diagonal are of equal length. **(6)**

c) With usual notation and figure show that **(6)**

$R_\theta=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$ and $\rho_\theta=\begin{pmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{pmatrix}$.

Q-6 **Attempt all questions** **(14)**

a) Solve the system of equation by Cramer's rule 2x+y=0 , 3y+z=1, 4z+x=2 . **(7)**

b) **(7)**

$$If\ A=\begin{bmatrix}1 & 5 & 0 & 0\\ 2 & 0 & 8 & 0\\ 3 & 6 & 9 & 0\\ 4 & 7 & 10 & 1\end{bmatrix}$$

Then compute detA using column vectors and inner product.

Q-7 **Attempt all questions** **(14)**

a) If x=(x_1, x_2, x_3) and $y = (y_1, y_2, y_3)$ then show that **(6)**

$$x\times y = \begin{pmatrix}x_2y_3 - x_3y_2\\ x_3y_1 - x_1y_3\\ x_1y_2 - x_2y_1\end{pmatrix}.$$

b) Show that $\det\begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix} = a_{11}a_{22} - a_{12}a_{21}$. **(4)**

c) What is r-linear map? Show that the map f : V× V →R where f(x , y) = $\langle x , y\rangle$ is bilinear map. **(4)**

Q-8 **Attempt all questions** **(14)**

a) State and Prove Caley-Hamilton theorem. **(5)**

b) Reduce the equation $11x^2 + 6xy + 19y^2 - 80 = 0$ into standard form. **(5)**

c) Write only the standard equations for the following conics and quadrics. **(4)**

(1) Imaginary ellipse (2) Hyperboloid of one sheet